**Leaf Energy Balance**

August 31, 2021

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Quadratic Solutions for Leaf Temperature and Latent Heat Exchange

Isothermal Energy Balance

One Sided Leaf

August 31, 2021

Taylor expansion Series



Linearization of Clausis Claperyon, saturation vapor pressure-temperature function, es(T)





Linearization of Stefan Bolzmann, T4



Solve for set of coupled energy balance equations, one Sided case

Sensible heat flux



Latent heat flux







Isothermal radiation budget, single sided leaf









































Substitute gw with gs and gb:







The leaf energy balance can also be used to derive a quadratic equation for latent heat exchange (E; W m-2):

*a LE2 + b LE +c =0*

We can solve directly for latent heat flux density



Linearize with 2nd order Taylor Expansion



I have second order terms of 

Substituting

Use 1st order expansion of Ts for LE







Multiply LE by 



The squared term is:



Substitute squared term and Simplifying:



Organize in terms of LE2 LE and intercept





Separate out a, b and c for quadratic solution







Compute Latent Heat Exchange

1: **Hypostomatous** leaves have stomata on one side. Their two-sided net radiation balance is:







Latent heat flux











1st order expansion





The leaf energy balance can also be used to derive a quadratic equation for latent heat exchange (E; W m-2):

*a LE2 + b LE +c =0*

We can solve directly for latent heat flux density



Linearize with 2nd order Taylor Expansion



Substituting

Use 1st order expansion of Ts for LE







Multiply LE by 



The squared term is:



Substitute squared term and Simplifying:



Organize in terms of LE2 LE and intercept





Separate out a, b and c for quadratic solution







Leaf Temperature

2nd order expansion

























Substitute gw with gs and gb:







1: **Amphistomatous** leaves have stomata on both sides. Their two-sided net radiation balance is:

Basic Equations











For Amphistomatous leaves, the stomatal resistance or conductance is defined as rs,leaf, which is the sum of the parallel resistors of the top and bottom. If we assume rtop equals rbottom





Also the boundary layer and stomatal resistances of the top and bottom are in series and rbottom equal rtop equal rstom.







Be careful and not to put factor of 2 twice, in gw and in LE..



Solve for Surface Temperature, Ts

From Ohm’s Law



From Energy Balance of two-sided leaf by simplifying common denominator





Taylor’s Expansion for T4

1st order







2nd order







Solve for delta T, as the equation above shows that delta T is a function of delta T



Insert Term for Latent Heat Exchange







Set terms to zero



Re-arranged to form Quadratic Equation in terms of Ts-Ta











The leaf energy balance can also be used to derive a quadratic equation for latent heat exchange (E; W m-2):

*a LE2 + b LE +c =0*

We can solve directly for latent heat flux density





Just to make sure I don’t miss any multiplication factors and signs, I factored this out



Substituting

For Algebraic simplicity we will introduce a truncated version of the second order expansion of surface energy balance and temperature into the equation for latent heat exchange, for the amphistomatous leaf



Be careful if factor of 2 on gw don’t need to multiply again by 2



Multiply LE by 



The squared term in the numerator is:





Insert the squared term into the main equation





Reorganize the Main Equation and collect terms



Collect terms for the quadratic function









